The Pompeii *Auloi* Improved Data and a Hitherto Unknown Mechanism

Stefan Hagel

Zusammenfassung

Für zwei der berühmten vier Aulosrohre aus Pompeji werden verbesserte Messdaten gegeben und ihr Beitrag zur musikalischen Interpretation der Instrumente diskutiert. Darüber hinaus wird erstmals die Konstruktion des Mundstücks untersucht, das das Rohrblatt aufnahm, und als Stimmvorrichtung interpretiert.

Of the auloi and aulos fragments1 retrieved during the excavations at Pompeii and now stored in the National Archaeological Museum at Naples, only a group of four pipes has as yet been examined with the aim of establishing the musical scales on the basis of which they are plausibly structured. They bear the inventory numbers 76891 through 76894; for the sake of convenience I omit the leading digits and call them simply Pipes 1 through 4. In a recent study I have interpreted them on the basis of both old and recent photographs;² meanwhile, I had an opportunity to examine some of the items in detail together with Olga Sutkowska, carrying out exact measurements and taking photographs on which most of these measurements could be doublechecked. Consequently, it is now possible to give more precise results for one pair of pipes; this is done in the first part of the present contribution. In the second part I describe a hitherto unknown detail of the instruments' construction and suggest an interpretation of its purpose.

The four pipes in question consist of an ivory core with a metal encasing and rotating metal 'sleeves' or 'rings', made alternately of silver and of copper alloy; the layers are constructed in overlapping sections. The sleeves usually cover or uncover a single finger hole; in a few instances, two holes: in these cases, only one of the two could be open in any given position of the ring, while it was also possible to close both at the same time. The number of finger holes per pipe varies from ten to nineteen. It is clear that the production of instruments of this type demands craftsmanship of the highest quality; structurally, however, the mechanism of these particular pipes is relatively straightforward, since it does not involve sections with three metal layers and internal rotating elements.³

There is no doubt that the pipes were played in pairs, so that two of them effectively formed a single instrument of the type the Greeks called aulós and the Romans tibia. The position of small metal knobs by which the sleeves were operated, or soldering traces where the actual knobs are lost, unambiguously assign each pipe to either the left or the right hand: Pipes 2 and 4 were held in the left hand, Pipes 1 and 3 in the right. In my previous study I suggested that they might have been played in various combinations, although Pipes 2 and 3 clearly complement each other especially nicely. On closer inspection, I am now inclined to think that we might instead be dealing with two distinct instruments, composed of Pipes 2 + 3 and Pipes 1 + 4, respectively, but a final decision must await thorough analysis of the latter pair, which we have not yet been able to carry out.4

Be that as it may, it seems clear that the set of instruments in some way belongs together, not just because they were found together and are of a very similar make that sets them apart from all other published fragments – as regards the use of materials, the diameters, and the specific shape of the

¹ On the *aulos* in general, the most typical wind instrument of the ancient Mediterranean, usually consisting of two independent reed-blown pipes, cf. Howard 1893; West 1992, 81–107.

² Hagel 2008; cf. Hagel 2010a, 351–361; Earlier studies include Gevaert 1881, 295–296. 645–647 (criticized by Howard 1893, 55 n. 1); Howard 1893, 51–55 (with experiments); Curtis 1914, 102 (based on flawed figures); Letters 1969 (raw calculation based on Howard's measurements and erroneous assumptions about mouthpiece lengths: cf. West 1992, 97 n. 81).

³ For such a type of mechanism cf. Olga Sutkowska's contribution to this volume.

⁴ When we worked in Naples, we were only able to see Pipe 4 for a short time and could not examine its parts as it was prepared for shipping; Pipe 1 was on exhibition.

parts close to the mouth of the player – but also because their evaluation from a musical point of view suggested that they share a common pitch standard and tonal paradigm. It emerged that the pipes were especially suited to a set of ancient keys ranging from the natural 'Lydian' (including modulation to Hyperlydian) in the direction of the 'sharp' keys up to 'Iastian', which is just the core of the range that an ancient source attributes to the *aulos* and also coincides with the keys used by the bulk of extant ancient Greek musical documents (no Latin scores survive). All this suited our knowledge of ancient music very well, with a single exception: following my interpretation, the pitch of the instruments was surprisingly high.

There is another point which puzzled me a lot in the course of my first evaluation (although I think I have hushed it up successfully in the publication). Since the reeds of the instruments have not survived (and very probably were not attached to the pipes when they became interred in the course of the Vesuvius catastrophe), it is necessary to determine an approximate value of their optimal effective length by means of calculation.⁵ It turned out that while Pipes 1, 3, and 4 required satisfactorily uniform lengths (3.0 cm, 3.1 cm and 3.5 cm), Pipe 2 needed a much longer reed (4.5 cm). Surprisingly, this unwanted disparity was counterbalanced by another one: on Pipe 2, the part where the reed is inserted was shorter than on the other pipes by the same amount by which its required reed was longer, so that the overall length of insert plus reed seemed to have been similar, after all. I noted the fact, but could not account for it (cf. Fig. 5).

During our visit to the National Archaeological Museum of Naples we obtained new measurements for Pipes 2 and 3. These pipes, although now fragmented, are fortunately in a condition that in many cases makes it possible to align the pieces seamlessly (guided also by the old photographs); only a few parts now appear to be lost. The finger-hole positions, verified from photographs in the way I describe in my other contribution to this volume, are laid out in table 1. As regards Pipe 2, they are based on the assumption that Howard's distance from the lower end of the pipe to the centre of the lowest finger hole is correct; this assumption is of course somewhat hazardous, but nevertheless inevitable, since the corresponding parts of the instrument are lost and the old photographs are not decisive as regards the lower end of the pipe. Anyway, this merely concerns the calculation of the lowest note, gained from the pipe with all finger holes closed; the relative positions between the holes are not affected. The table also contains the other values required to calculate pitches: the overall length of the tube (subject to the reservations made above concerning the lower end of Pipe 2), the diameter of its bore (which is to all intents cylindrical), and its external diameter in the region of the finger holes (which is also practically cylindrical, apart from the fact that the corrosion of the copper alloy parts, although cleaned, in many points still slightly enlarges the diameter). It almost goes without saying that the bulb close to the top of the instrument includes a cylindrical bore that continues that of the main part. It is only at the very top that the bore widens to receive the reed; we examine this part of the construction below. As for the finger holes, which are mostly elliptic in shape, their longitudinal and transversal diameters are detailed ($\emptyset_1 \times \emptyset_1$);⁶ the distance values refer to their centres.

In order to establish the pitches that would have been produced from each of the finger holes, and the intervals between them as well as possible scales formed by these, one needs to know the proper 'effective length' of the reeds that were attached to the instruments. Since these are lost, one must experiment with possible values in order to identify the optimal configuration; I have done this with the dedicated software I describe in a contribution to an earlier volume of this series.⁷ There is indeed an unambiguous solution that gives both a maximum number of consonances (i.e., fourths, fifths and octaves, in accordance with the ancient view)⁸ within and between the pipes and meaningful scales. It requires an effective reed extrusion of 4.0 cm on Pipe 3 - half a centimetre more than had appeared from the old set of data; once more, the details for Pipe 2 must be postponed. The pitches (in hertz) as well as the intervals between adjacent notes (in cents) can be read from the screen snapshot in figure 4.

In order to count 'consonant' intervals, one must of course define an accuracy threshold. I am accustomed to accept calculated intervals that deviate from the theoretical ideal (498 cents for a fourth, 702 cents for a fifth and 1200 cents for an octave) by as much as 20 cents (a tenth of a tone): on the one hand, a mismatch of that size can still be counterbalanced rather easily by means of embouchure variation; on the other hand, no less an ancient musical writer than Ptolemy expressly regards a similar divergence (in fact, 21.5 cents) as practically negligible.⁹ All calculated unisons,

⁵ I have described the software I have developed for this purpose in Hagel 2004.

⁶ An elliptical finger hole has the same effect as a circular one with the same area, so that for pitch calculation one can work with an 'effective' diameter of $\mathcal{O}_{\text{eff}} = \sqrt{\mathcal{O}_1 \mathcal{O}_1}$,

⁷ Hagel 2004, 380–381; cf. also Hagel 2010b, 71.

⁸ These, and their combinations with the octave, count as sýmphōnos in ancient theory, the two constituent pitches being perceived as blending into a uniform whole when sounded simultaneously, while those of other intervals were still perceived as independent (for the ancient sources cf. Scheltema 1932–1933, 241–242).

⁹ Ptolemy, *Harmonics* 1.16, page 39.19–22; 40.1–6 Düring.

fourths, fifths and octaves accurate to 20 cents are listed in table 2.

The list holds 73 items – obviously there were quite a lot of consonances of which the player could avail himself (which in this case seems more likely than 'herself'). At the same time, a comparison with earlier evaluations further demonstrates the validity of the method: the more accurate the measurements, the better the musical results, even though the latter consist in mere computer calculations. In figure 1, our new consonance count is juxtaposed with that of my earlier study, which was based on photographs and Howard's figures, and with a count based on the optimal configuration for the figures originally given by Howard. The full harmonic coherence of the instrument only emerges from the most accurate data, a fact which proves as well that apart from unisons and octaves, consonant fifths and fourths were pivotal in the design of these instruments, and, therefore, played an essential role in the music of the period.¹⁰

A similar point emerges with regard to the question of the absolute pitch to which the instrument is tuned. The conclusion of my former study, that the principal 'keys' that the set of four pipes were designed to play range from Lydian to Iastian, also holds true for the re-evaluated instrument consisting of Pipes 2 and 3; within the said range of keys, this instrument exhibits a special prevalence of the Hyperiastian and Iastian; the lowest part of the pipes seems to have been built exclusively for these (cf. Fig. 2).¹¹

Now while the question of playable keys can thus be answered unequivocally, as I have already said my original calculations vielded an absolute pitch that seemed a bit too high: about a sixth of a tone higher, in fact, than what modern scholarship agrees would have been the upper limit of the ancient 'standard':12 the pipes' 'Lydian mése' (which is a convenient reference note), whose pitch equivalent is conventionally given as about between modern b flat and b, seemed to sit between b and cinstead. Once more this result was due to deficient measurements: with our corrected data, the divergence vanishes and the pitch of the pipes now appears to be identical with the agreed upper limit of ancient 'standard pitch' (see Fig. 3: Howard's old figures for the finger hole positions would again have led to a much more pronounced error).

Finally, we may now turn to the problematic reed of Pipe 2, which still needs to be oversized in order to make up for the unusually short insert part, as shown in figure 5. The same figure, together with the two following ones, also illustrates the typical structure of the highest part of the pipe. The white bulb forms the only element of the instruments that is not encased in metal. In a smooth curve, its lower end tapers down beneath the diameter of the main tube, then expands exactly to this diameter at the point where it is connected to it by means of a spigot. A small distance below the thinnest point a decorative incision runs round it. On its other end, in contrast, the curvature breaks abruptly into a cylindrical part. Originally this was once more clad in copper alloy, as its discolouration proves (a fragmentary pipe where this part of the encasing is extant is shown in figure 6). Above the cylinder we find the cone, in which the reed is inserted; it flares out in an elegant twofold motion. Incised lines probably helped in gluing the metal encasing onto it, which was further secured by folding its end over the higher end of the cone into a recession obviously made exactly for this purpose (Fig. 7; its depth of 0.75 mm gives us a good estimate of the thickness of the encasing). Internally, the main bore continues up to about 14.9 mm below the end, where it widens in a step from ca. 8.2 mm to ca. 10.5 mm, the external diameter of the reed (perhaps including a winding of waxed thread).

This structure is not made out of a single piece. From figure 7 one may suppose that the clear-cut step is actually created by inserting a smaller tube into a larger one; also visible there, about the centre of the smaller tube, is a line running around the internal wall, raising the suspicion that this is the meeting point of two smaller tubes. Closer inspection shows that this is indeed the case: at the lower end of the bulb of Pipe 2 the spigot is broken off in a way that exposes its nature as the prolongation of an internal cylindrical tube, as seen in figure 8. The same photograph also shows the cone part taken off, exhibiting the second inner tube protruding from the bulb in the other direction.

It is not difficult to see why such a 'complicated' construction in four parts would have been preferred over a 'simple' one. First of all, smaller pieces of ivory (or bone) are easier to come by, and certainly cheaper. Secondly, although drilling the small tubes and subsequently turning them down on the lathe to a wall thickness of hardly more than a millimetre is a delicate task, it still involves less trouble than producing the entire unit from a single piece: after all, in the case of the spigot, it would still be necessary to manufacture a length of thin

¹⁰ The optimisation for fifths and fourths yields a count of 52 intervals of this kind. For comparison, if the theoretical reed extensions are optimised not for fifths and fourths, but for pure major and minor thirds (still in addition to unisons and octaves), one obtains a count of only 42 thirds within the same threshold, but still 50 fifths and fourths. (there is however little doubt that more pure major and minor thirds could be produced by embouchure adjustment).

¹¹ For the details of this diagram cf. Hagel 2010a, 354–355.

¹² On the arguments for ancient absolute pitch cf. West 1992, 273–276; for additional evidence, Hagel 2010a, 68–95.

tube; but with the bulky rest attached to it in the process of turning, the risk of breaking it is considerably higher.

The internal step down to which the reed goes is also most easily realised by means of such a construction made of telescoping tubes. However, when the corresponding parts of Pipe 2 are taken apart, it becomes clear that here the projection of the internal tube into the cone does not leave enough room for the reed (Fig. 9): a mere 4.5 mm (as compared to the 14.9 mm in Pipe 3) is clearly insufficient to anchor it, and also entirely unparalleled.¹³ Therefore, the problematic insert part of Pipe 2 in its present state

- is shorter by about a centimetre in comparison with the corresponding parts on Pipes 1, 3 and 4,
- would require a reed that protrudes from the insert for about a centimetre more than on the others, while
- extending into the insert by about a centimetre less.

In other words, the reed would actually have the same dimensions, starting and ending at the same points as on the other pipes - it is only the insert that stops short of the others by one centimetre. The conclusion is inevitable that the problematic part is in fact damaged. The lower part of the cone element has broken away, probably by squashing the cone towards the bulb. The result of such an accident is predictable: while the thicker part of the cone may survive such a procedure intact, its lower delicate part would explode and break away. If the entire piece is not split, the lower portions would break off roughly symmetrically, because the internal tube acts as a guide so that bulb and cone cannot be tilted in relation to each other. This kind of damage would typically occur if the instrument were dropped vertically, mouth end downward. It had so far escaped former researchers' eyes because the broken end is still largely covered by the metal, so that the unsmooth connection of the core is concealed. Consequently, the insert part of Pipe 2 is to be restored in analogy to that of Pipe 3, as done graphically in figure 10. The calculated required effective reed extrusion is now 3.6 cm, which is satisfactorily similar to the 4.0 cm for Pipe 3.14

It remains to be explained why the lower end of the cone was so exceptionally fragile. In fact it was clearly more delicate than it needed to be on the assumptions we have made so far, namely that putting the structure together from several parts was the cheapest and easiest option. This does not account for a detail which we have not yet addressed: the cylindrical section between bulb and cone is not manufactured as a part of either the one or the other, but about half of it had no fewer than four

layers. The slender internal tube is the innermost element, followed by a short cylinder extending from the bulb, and above that a very thin cylinder extending from the cone, which in the end was covered in metal. One might think that this particular arrangement was supposed to enforce the structure of the whole thing, so that potential strains would not only rest on the innermost tube. Yet this does not account for the fact that the outermost layer of ivory is as thin as it actually is - much thinner than the layer below it (cf. the splintered away parts of Pipe 4 in figure 11 and Pipe 3 in figure 12, where a drawing of the construction is also given). After all, the thing has broken at this point. Among all the auloi and aulos fragments we know, most of them sporting spigot and socket connections, it is only here that parts of such markedly different thicknesses are connected. Now, if we can rule out that this was done for higher stability, it seems that the motivation for this surprising detail must be found in the realm of aesthetics. Yet details of layering would hardly influence the sound, and since, once put together, the entire section was covered by the outermost layer, there is also no gain as regards visual appearance. Unless, that is, the parts were not all permanently fixed together.

If the possibility is accepted that they were meant to move against each other, the peculiarities of the construction become aesthetically functional: when the cone is pulled outwards by up to 6 mm, the extreme slenderness of its lower part ensures that no really noticeable step is produced in the outline of the instrument; in fact, the resulting step may not exceed a millimetre by much. Admittedly, there is still the difference in colour, because a ring of the underlying material is laid bare; but this additional stretch of polished white between shining bronze and more polished white would hardly stand out conspicuously.

What might be the benefit of such a mechanism? When starting a performance on the *aulos*, the principal problem is bringing the pipes in tune with each other (which at the same time ensures correct scales, if the instruments are well built). Slight adjustments can be made by exerting lip pressure on different parts of the reeds – but this also affects the volume, so that a good balance between the pipes would possibly have to be traded against a better tuning. In order to optimise both

¹³ Other reed inserts of measurable depth include: the Louvre *aulos*: 13.8 mm and 14.6 mm (the proportions of the drawing in Bélis 1984, 114, are misleading); the Berlin *aulos*: 14 mm (Berlin Egyptian Museum inv. 12462; cf. Hagel 2010b); Lecce National Museum inv. 12528(7): 19.9 mm.

¹⁴ Anyway, a small disparity is to be expected from the remark by Theophrastus, *Historia plantarum* 4.11.7, that the two reeds made from adjacent sections of a stem of cane formed a pair, but were not exchangeable between the pipes.

parameters at once, either the opening of the reed or its extrusion from the pipe can be altered. The former is hardly an option in a performance situation. It is usually possible to realise the latter by pushing the reed further in or pulling it out a bit. Yet these procedures are also not without hazards. Apart from the danger of damaging its delicate blades (after all, there is little else where to handle it), there is always a chance of impairing the airtightness of its connection to the tube, making the pipe unplayable. Under the strict conditions of the ancient professional musical world, performers could be expected to embrace a mechanism that enabled them to tune their instruments quickly and safely. And that is exactly what the insert part of the Pompeii pipes would allow one to do (cf. Fig. 13). In any case, a span of 6 mm is certainly sufficient to make necessary last-minute corrections once the reed has been prepared accordingly.¹⁵

So far this is of course a hypothesis, to be overthrown if a better explanation for the startling thinness of this layer can be found. There is however another detail which I think lends this hypothesis the highest credibility. Up to now we have not dealt with two important, technically related questions. One is that of friction: how can it be ensured that the fitting between the tubes that move against each other is neither too high, which would render the mechanism inoperable (the greater the force that must be exerted, the more difficult it becomes to make small adjustments), nor too low, meaning that the distance might change inadvertently during playing? The second issue concerns air tightness: how could the tubes fit within each other so exactly as to avoid leakage? With only two tubes of bone or ivory and nothing in between, it would be very doubtful that both conditions could be met at the same time. The natural solution, adopted on numerous types of woodwind instruments throughout history, is to insert a layer of a softer, slightly yielding material between the rigid tubes as a seal: cork, for instance, or, much more easily, a winding of waxed thread. If this sealing substance is not glued onto one of the tubes, it is essential to keep it at some distance from the end of the internal tube. Otherwise there is the risk that, after a number of inward and outward adjustments, some parts of it are drawn over the edge of the tube and enter the internal bore, with detrimental consequences for the building of a stable oscillatory regime. Less importantly, one will also maintain some distance to the other end, so that a possible loose end cannot easily move into the space between the end of the external tube and the wall which faces this end, which would make it impossible to push them together entirely. As can be seen in figure 14, the protruding internal tube of Pipe 2 exhibits a discoloration which shows all the predicted features: it is clearly not induced by metallic corrosion, but due to another, unknown substance; and it stops short at some distance from the edge of the tube, and at a smaller distance from the step at the other side. At each of these ends we find an incised line, without doubt marking its intended boundaries, and probably also meant to prevent whatever it was that left the colour traces from shifting. This would be in accord with a winding of thread: if the first and the last loop were tightly fitted within these grooves, they would have been less likely to move and would thus prevent the entire winding contained between them from shifting. Nevertheless, and this also substantiates the waxed-thread interpretation against that of a glued-on seal, at the upper end the staining obliquely extends beyond the incision, just as a loop of thread does when pulled sideward.

Combining all this evidence, I am rather confident that an interpretation of the complex bulband-cone construction on the examined *auloi* as a tuning mechanism is justified. Its material reconstruction, while certainly desirable in principle, cannot be expected to contribute much to our understanding of its working (as opposed to the process of its manufacture), since the applicability of waxed thread is well known, while there seem to remain no open questions about the operability of the rest of the system, once it is construed in the way we have examined.

Acknowledgements

My warmest thanks go to the Soprintendenza Speciale per i Beni Archeologici di Napoli e Pompei, and especially to Dr. Marinella Lista and her team at the Museo Archeologico Nazionale di Napoli for making our research possible, and to Tosca Lynch for editing my style.

¹⁵ A increase of 6 mm in overall effective length theoretically decreases the pitch of the highest finger hole of Pipe 2 by about 40 cents, its lowest pitch by about 17 cents; for Pipe 3, the corresponding values are 46 cents and 19 cents. Actually the gain should even be somewhat larger, because drawing the cone outwards not only results in the internal cavity being prolonged but also creates a short section of a larger diameter (by ca 1.7 mm) that would add to the decrease in pitch.

Bibliography

Bélis, A.

1984 Auloi grecs du Louvre, Bulletin de Correspondance Hellénique 108, 111–122.

Curtis, J.

1914 The Double Flutes, Journal of Hellenic Studies 34, 89–105.

GEVAERT, F. A.

1881 Histoire et théorie de la musique de l'antiquité II (Gent).

Hagel, S.

- 2004 Calculating Auloi the Louvre Aulos Scale, in: E. Hickmann – R. Eichmann (eds.), Studien zur Musikarchäologie 4, Orient-Archäologie 15 (Rahden/Westf.), 373–390.
- 2008 Re-evaluating the Pompeii Auloi, Journal of Hellenic Studies 128, 52–71.
- 2010a Ancient Greek Music. A New Technical History (Cambridge).

2010b Understanding the Aulos Berlin Egyptian Museum 12461/12462, in: R. Eichmann – E. Hickmann – L.-Ch. Koch (eds.), Studien zur Musikarchäologie 7, Orient-Archäologie 25 (Rahden/Westf.), 67–87.

Howard, A. A.

1893 The αὐλός or Tibia, Harvard Studies 4, 1–63.

LETTERS, R. J.

1969 The Scales of Some Surviving Auloi, Classical Quarterly 19, 266–268.

SCHELTEMA, H. J.

1932–1933 De antiphonia, Mnemosyne n.s. 60, 239–253.

West, M. L.

1992 Ancient Greek Music (Oxford).

Pipe 2				
length: c. 541 mm external diameter: 12.8–13.9 mm bore diameter: 7.7–8.0 mm				
	finger holes			
nr.	size (mm) $Ø_1 \times Ø_t$	distance from lower end of pipe (mm)		
1	5.9 × 5.9	63.5		
2	8.2 × 7.3	93.0		
3	7.5 × 7.1	145.0		
4	6.9×6.4	169.6		
5	7.5×6.4	188.0		
6	8.8×6.6	212.8		
7	8.7 × 6.6	254.5		
8	7.0×7.0	269.1		
9	7.4 × 7.5	285.3		
10	7.4×7.4	301.6		
11	7.9×7.7	329.1		

Pipe 3			
length: 487 mm external diameter: 13.0–13.7 mm bore diameter: 8.0–8.2 mm			
finger holes			
nr.	size (mm) $Ø_1 \times Ø_t$	distance from lower end of pipe (mm)	
1	6.0 × 5.9	15.1	
2	7.6 × 6.3	44.8	
3	7.5 × 6.3	98.6	
4	5.2 × 5.2	123.0	
5	7.1 × 6.8	139.3	
6	6.0 imes 6.0	162.9	
7	6.1 × 6.1	184.1	
8	7.2×6.5	204.5	
9	5.9 × 6.0	224.2	
10	6.7 × 6.1	237.2	
11	7.0×7.0	250.9	
12	7.9×7.0	264.0	
13	7.5 × 6.3	283.2	
14	7.2 × 6.2	311.6	

Tab. 1 Finger hole measurements.

Between Pipes 2 and 3			
interval	hole on 2	hole on 3	dev.
1:1	1	0	+17
1:1	1	1	+18
1:1	2	2	+4
1:1	3	3	+12
1:1	4	4	+1
1:1	5	5	+3
1:1	6	6	+16
1:1	7	8	+13
1:1	8	9	+0
1:1	9	10	+4
1:1	10	11	+9
1:1	11	13	+14
2:1	0	10	-15
2:1	1	12	-10
2:1	11	2	-13
2:1	2	13	-3
3:2	1	7	-1
3:2	7	2	+14
3:2	2	8	-2
3:2	3	10	-1
3:2	9	3	-8
3:2	4	11	-15
3:2	10	4	-7
3:2	5	12	-4
3:2	11	6	+2
3:2	6	13	+1
3:2	7	14	-3
4:3	0	3	-3
4:3	5	0	+10
4:3	1	5	-4
4:3	6	2	+0
4:3	2	6	-19
4:3	3	8	+13
4:3	7	3	+14
4:3	4	9	+7
4:3	8	4	+6
4:3	5	10	+17
4:3	9	5	+18
4:3	10	6	+16
4:3	6	11	-9
4:3	11	8	-15
4:3	7	13	-13
4:3	9	14	+19

Within Pipe 2				
interval	hole 1	hole 2	dev.	
2:1	0	9	-11	
2:1	2	11	-17	
3:2	2	7	+10	
3:2	3	9	+4	
3:2	4	10	-6	
3:2	6	11	-14	
4:3	0	3	-14	
4:3	1	5	-6	
4:3	2	6	-3	
4:3	4	8	+7	
4:3	6	10	+0	

Within Pipe 3				
interval	hole 1	hole 2	dev.	
2:1	0	12	+7	
2:1	2	13	+1	
2:1	3	14	+12	
3:2	0	7	+16	
3:2	1	7	-19	
3:2	2	8	+1	
3:2	3	10	-12	
3:2	4	11	-16	
3:2	5	12	-6	
3:2	6	13	+17	
3:2	8	14	+10	
4:3	0	5	+13	
4:3	2	6	-16	
4:3	3	8	+2	
4:3	4	9	+6	
4:3	5	10	+14	
4:3	6	11	+7	
4:3	7	12	-9	
4:3	8	13	0	

Tab. 2 Calculated consonant intervals. Finger holes are counted from the lower end of the instrument, with '0' representing the tube with all holes closed. Intervals are labelled by ratios of frequencies: 1:1 = unison; 2:1 = octave; 3:2 = fifth; 4:3 = fourth; 'dev.': calculated deviation from pure interval (up to ± 20 cents).



Fig. 1 Concords increasing with accuracy of measurements (made by S. Hagel).



Fig. 2 Notes and scale (*tónoi*) fragments playable on the instrument consisting of Pipes 2 and 3. Modern note names indicate relative diatonic pitches (chromatic notes in parentheses) (made by S. Hagel).



Fig. 3 Pitch deviation (in cents) from the accepted ancient standard pitch decreasing with the accuracy of measurements (made by S. Hagel).



Fig. 4 Optimal reed configuration for Pipes 2 and 3 (image by S. Hagel). From left to right: calculated pitches (Hz), intervals between adjacent notes (cents),

- distance from the lowest note (cents),
- modern note equivalents (based on a = 440 Hz; deviations in cents),
- ancient note equivalents (the Lydian *hypátē* set to 185.4 Hz; approximate deviations in cents), approximate consonant intervals between the pipes (size in cents),
- a possible diatonic scale (in relative pitch).



Fig. 5 Bulbs, reed inserts and calculated optimal effective reed lengths for Naples National Museum Inv. 76891-4 according to Hagel 2008 (photographs courtesy of the Naples National Museum; montage by S. Hagel).







Fig. 7 Internal view of bulb plus reed insert of National Museum Inv. 76893 (Pipe 3) (photograph by S. Hagel; courtesy of the Naples National Museum).



Fig. 8 National Museum Inv. 76892 (Pipe 2), upper end of main part; bulb with broken inner tube; reed insert (photograph by S. Hagel; courtesy of the Naples National Museum).



Fig. 10 Bulbs, reed inserts and calculated optimal effective reed lengths with a restored insert of Pipe 2 (photographs courtesy of the Naples National Museum; montage by S. Hagel).



Fig. 11 Bulb and reed insert of Naples National Museum Inv. Nr. 76894 (Pipe 4) (photograph courtesy of the Naples National Museum).



Fig. 12 The internal construction of the bulb + reed insert part of Pipes 2 and 3, and presumably also Pipes 1 and 4 (drawing by S. Hagel; photograph courtesy of the Naples National Museum).



Fig. 13 Fine-tuning the effective reed extrusion by shifting the insert cone (drawing by S. Hagel).



Fig. 14 Internal tube, with discolouration, of Naples National Museum Inv. Nr. 76892 (Pipe 2) (photograph by S. Hagel; courtesy of the Naples National Museum).